# An Ethnologist's Guide to Stronger Math Instruction and Achievement

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This article incorporates lengthy excerpts from the chapter entitled "East Asian Primary Schools, Part II: How Mathematics Lessons Are Delivered," from Grove's book, *A Mirror for Americans: What the East Asian Experience Tells Us about Teaching Students Who Excel*, published in 2020 by Rowman & Littlefield.

## Preface

As an American ethnologist of education, my work is like that of a sports team scout. I compare the pedagogical practices in other societies with each other and with ours here in the U.S. My objective is to identify practices that most often mold students who excel academically. How do those teachers think about instruction? What are they doing in their classrooms? I bring my findings back to my home team here in the U.S., hoping they'll help my fellow educators up their game, so to speak.

Students who excel are often molded in East Asia. Teaching there has been massively researched.

Recently I received an *Education Week* publication entitled, "An Educator's Guide to Stronger Math Instruction and Achievement."[1] Its short introduction noted that "*Education Week* reporters pored through the research to find the best practices to teach students the math skills they need, and to offer practical tips and strategies for math teachers." I wondered whether the research those reporters pored through included any of the hundreds of studies completed in East Asia.

Although I can't be sure, I cannot find in this publication an indication[2] that any of the research considered by the reporters originated in East Asia. If they included little or none of it, that's really disappointing because East Asian math students consistently outperform their American peers.[3]

To compensate for what I suspect was the reporters' oversight of a huge and compelling body of research, I offer herein my "Ethnologist's Guide to Stronger Math Instruction and Achievement."

## A Sample Comparison

In the publication cited above, one article was entitled, "Math Gets Progressively More Abstract. Here's How to Help Students Keep Up." I'm not a math teacher, but I do know that getting students to calculate abstractly is a big hurdle for American teachers. So what do we know about East Asia?

Consider the "doorbell problem": Sally is having a party. The first time the doorbell rings, one guest enters. The second time the doorbell rings, three guests enter. The third time the doorbell rings, five guests enter. Each time the doorbell rings, the number of guests entering is two larger than entered on the previous ring. *How many guests will enter on the fifth rin g?*

The "doorbell problem" was given by researchers to pupils in the U.S. and China[4] to compare how they think about math and how they go about solving problems. As every math teacher knows, there are two ways of approaching this problem: "concrete" and "abstract/symbolic."

**Concrete:** That's easy! How many guests enter on ring 5? That would be 1+2+2+2+2 = 9.

When there are only a few rings, this "concrete" or "counting" solution process works well. But as the number of rings increases – How many guests enter on ring 55? – more and more children and adults need to make drawings or notes, which can yield the correct answer but (a) is labor intensive, and (b) fails to use a strategy that efficiently solves similar but larger problems.

**Abstract/Symbolic:** The number of guests who enter on a given ring equals twice the ring number, minus one (because on the first ring, one guest entered instead of two). To use symbols, we'll say that $g$ is the number of guests and $n$ is the ring number. So we can state that $\boldsymbol{g = 2n - 1}$, an "abstract" formula that works with small *and large* problems. How many guests enter on ring 5? That would be $g = (2 \times 5) - 1$; $g = 10 - 1$; $g = 9$. How many on ring 55? Without being labor intensive, that would be $g = (2 \times 55) - 1$; $g = 110 - 1$; $g = 109$.

*On one of the rings at Sally's party, 99 guests enter. Which ring?*

Answering via the concrete method is very labor intensive. But if we use the formula above, we can quickly determine the answer: $99 = 2n - 1$; adding 1 on both sides of the equal sign yields $99 + 1 = 2n$; $100 = 2n$; $50 = n$.

Using this 99-guests problem, researchers found a difference between Chinese and American pupils that other studies also found. Chinese pupils are more likely to approach a challenging problem on a conceptual level – i.e., to apply an abstract/symbolic strategy – which amplifies their capacity both to efficiently attain the correct answer, and to recognize that more than one strategy can attain the correct answer.[5]

**Solution Strategies: Doorbell Problem with 99 Guests**

| Solution strategy | CHINESE PUPILS | | AMERICAN PUPILS | |
|---|---|---|---|---|
| | Percent trying to apply this strategy | Of those who tried, percent succeeding | Percent trying to apply this strategy | Of those who tried, percent succeeding |
| **Concrete**<br>Counting that applies only to *this* problem, not others | 29% | 37% | 75% | 46% |
| **Abstract**<br>Developing a formula that applies to similar problems | 65% | 98% | 11% | 60% |

Source: Jinfa Cai & Victor Cifarelli (2004). Thinking mathematically by Chinese learners: A cross-national perspective. *How Chinese Learn Mathematics: Perspectives from Insiders*. Based on portions of Table 4, p. 88.

## A Fifth Grade Math Lesson in Taiwan

One study provided this account of a fifth-grade teacher in Taiwan who was presenting a geometry lesson to a large class.

> The teacher drew attention to an unusually shaped figure she had neatly drawn on a small blackboard before the class began. She asked how they might go about finding the area of a shaded region, saying, "I don't want you to tell me what the actual area is, just tell me the approach you would use to solve the problem. Think of as many different ways as you can to determine the area shaded with yellow chalk."
>
> She allowed the pupils several minutes to work in small groups and then called on a child from each group to describe the group's solution. Each child came to the front of the room to make his or her statement while pointing to the figure. After each description, many of which were quite complex, the teacher asked members of the other groups whether the procedure described could yield a correct answer.
>
> After several different procedures had been suggested and evaluated by the class, the teacher moved on to a second figure and repeated the entire sequence of steps. Neither teacher nor pupils attained a solution to either problem until all of the alternatives had been discussed.
>
> The lesson ended with the teacher's affirming the importance of seeking multiple solutions – to life's problems as well as to math problems.[6]

This lesson wasn't only described in writing. It was also videotaped and included in a video about East Asian teaching produced by the University of Michigan. That video, entitled "The Polished Stones: K–5 Math Achievement in Taiwan and China," is still available – and relevant.[7]

To watch this video, go to youtube.com/watch?v=Tpr6Q2FsJyE. The entire video is 35 minutes long; the lesson mentioned here begins at the 12-minute mark. Be sure to notice the size of the class.

## General Features of East Asian Math Lessons

Students in East Asia aren't superior only in math. Yet how they learn math has drawn a disproportionate amount of interest. Here's why:

The technical details of math never vary, and the way in which math is written (using numbers and symbols such as % and √) is nearly universal. But *how* math is taught varies from place to place. So math affords a unique opportunity for researchers to compare "the how" of teaching across cultures, one that avoids the complications resulting from variations in the contents of "the what" being taught. Consider:

What is the area of a triangle with such-and-such dimensions? The answer is identical everywhere, as are efficient methods of finding it. This is true of no other subject except perhaps physics or chemistry.

The effort to understand "the how" of East Asian math teaching has yielded insights of two types: *general features* and *specific strategies*. We'll begin with these general features:

- The stance of the teacher vis-à-vis the pupils
- The handling of pupils' errors in reasoning
- The lesson's pace and degree of focus
- The nature of classroom verbal interactions

**The Stance of the Teacher vis-à-vis the Pupils**[8]

"Stance" refers to the attitude, mental and emotional, that the teacher takes toward the youngsters arrayed before him or her – *and* toward the knowledge those youngsters are learning.

In Chapter 6 of *A Mirror for Americans*, I noted that teachers in East Asia are "directive" but rarely "authoritarian." For example, during math lessons, the teacher is not the one who authoritatively decides whether a pupil's solution or answer is correct. That's unlike the usual practice here in the U.S., where teachers usually *are* the arbiters of correct and incorrect: When a pupil replies to a question, it's virtually always the teacher who pronounces the answer right or wrong.

Recall the description (and video) of that fifth-grade geometry lesson. After each group's procedure for finding the area was proposed, the teacher *asked members of the other groups* whether that procedure could yield a correct answer. East Asian teachers leave the analysis of correct or incorrect to their pupils (but ensure they get it right!).

With this stance in mind, let's revisit one of the analogies drawn in Chapter 5: the teacher as a virtuoso performer. Noting how often and how meaningfully pupils contribute to math lessons, two researchers suggested replacing the virtuoso analogy with *orchestra conductor*.[9]

They could have proposed *facilitator* or *coordinator*, but they stayed with the musical theme. Unfortunately, orchestra conductor doesn't work; conductors rehearse their orchestras until all the players are highly familiar with the conductor's vision for the musical piece to be performed. Teachers and pupils *never* rehearse classroom lessons, not even lessons that will be piloted for a group of observing fellow teachers under the Lesson Study process.[10]

**The Handling of Pupils' Errors in Reasoning**[11]

Errors in reasoning are inevitable in primary-school math classes. How are pupils' incorrect ideas dealt with by teachers in East Asia?

This question relates to an East-West cultural difference that was discussed in Chapter 3 of my book, *The Drive to Learn*: how people react to success and failure. Failure in the U.S. tends to be embarrassing, so people try to minimize or ignore it. Teachers assume that a pupil's faulty reasoning or incorrect answer will demotivate him, so they say nothing and move on to other pupils until they get the right answer.

Things could hardly be more different in East Asia. There, mistakes suggest a path for exploration. The teacher, without pronouncing an answer wrong, will ask the pupil how it was mathematically derived. The pupil explains. The other pupils listen; when asked, they chime in with one or more alternative derivations. Class discussion eventually yields the correct answer.

Here's the point: In East Asia, exploration of the reasoning that led to each error is an important contributor to pupils' conceptual grasp of whatever they're learning. Teachers' *directive facilitation* of the pupils' learning process is enhanced by analyses (by teacher and/or pupils) of the step-by-step reasoning that led to an incorrect answer.

There's no evidence that pupils whose errors are publicly dissected feel embarrassed. The spirit that prevails isn't "gotcha!" Rather, it's that we're all collaboratively involved in analyzing reasoning methods. So instead of being an embarrassment, failure usually is viewed as providing new guide-posts for finding effective ways to do something.[12] For pupils, it's all rather matter-of-fact:

Interviewer: What would happen if you answered incorrectly?

Student: Incorrectly? Just sit down and the teacher would ask another student to answer.

Interviewer: How do you feel then?

Student: Nothing. Wrong means wrong. This time wrong, next time it will not be wrong.[13]

**The Lesson's Pace and Degree of Focus**[14]

Some believe that the explanation for the academic superiority of students from East Asia is that, year after year, their lessons progress through the material faster than American lessons, enabling older students to know more.

Researchers discovered that the reverse is true. Asian classes proceed at a slower pace, a fact easily observed in math classrooms.

In one study, the number of topics covered during math lessons in a number of first-grade classes was counted. The average number of topics during the lessons in the U.S. was 4.17; the average in Taiwan was 3.55; the average in Japan was 2.35.[15]

The learning objectives of all the lessons were single-digit and multidigit operations, and place value: three topics. During the U.S. lessons, eight *other* topics were also discussed, including time, money, calendar, and (the second most-discussed topic) fractions. During lessons in Taiwan, three other topics were discussed. During the Japanese lessons, only two other topics were discussed.

So the American pupils are learning more math, right? Wrong. More topics are being *mentioned* in each class, but the pupils are having fewer opportunities to make progress toward mastering the handful of topics that are that lesson's objectives.

Shorthand ways of describing American and Japanese teaching methods have been proposed. Americans believe a teacher should subdivide a lesson into small steps so that most students will quickly grasp each one, after which the teacher moves to the next one. This strategy has been called *quick and snappy*. In Japan, pupils are expected to linger over the issues introduced by the lesson. Teachers pose thought-provoking questions and expect inquiring pupil-to-pupil exchanges that lead to long-answer resolutions. This strategy has been termed *sticky probing*.[16]

**The Nature of Classroom Verbal Interactions**[17]

Pupils in East Asian primary schools talk more during math classes than Americans. For example, a research team devised a way to measure the quantity of pupils' classroom talk. During lessons on triangles and fractions, Japanese pupils publicly spoke approximately twice as much as their American peers. Furthermore, the word count of Japanese pupils' statements was far greater than the length of the Americans' statements. All this talking slows the pace but strengthens the learning.

But what are they talking *about*? Another research team looked at fourth- and fifth-grade math lessons in China and the U.S., comparing the nature of lengthy classroom discussions. Here's what they found:

The Chinese teachers' questions usually prompted their pupils to explain their procedures ("How did you get that answer?"), to state their mathematical reasoning ("Why was that procedure appropriate?"), or to cite an applicable rule ("What's the rule governing that?"). They also used discussions to promote pupils' awareness of the underlying rules. The outcome was that pupils were often drawn to connect specific problems with math's fundamental regularities.

The American teachers' questions usually asked their pupils to think about the result of a computation ("What answer did you get?"). Their focus on calculation accuracy rarely stepped all the way toward making connections with underlying mathematical rules and regularities.

Here's an interesting finding from eighth-grade classrooms. One study looked at the topics of talk by students and teachers in the math classrooms of four East Asian cities and two Western cities (San Diego, CA, and Melbourne, Australia). The researchers discovered that discussions about *any topic whatsoever* occurred more frequently in the two Western cities. But when they counted *only* the occurrence of key mathematical terms – such as equation, co-planar, and hypotenuse – the East Asian classrooms, and especially those in Shanghai, were ahead of those in the West. Commenting on these findings, they wrote:

> Students in Shanghai had the opportunity to articulate their understanding of key mathematical terms through a structured process of teacher invitation and prompt that built upon the contributions of a sequence of students. Classrooms in Japan provided many instances where a student made the first announcement of a term without specific teacher prompting.[18]

## Visiting Similar Lessons in Japan and the U.S.

Before we consider the specific strategies of math teachers in East Asia, let's pause to look in on third-grade math lessons in the U.S. and Japan.

### An American Lesson Introducing Fractions[19]

The teacher announced that today's lesson concerned "fractions." She defined a fraction and wrote a few on the board, in each case pointing out the "numerator" and "denominator." A quick and snappy review followed: "What do we call this?," she asked. "And this?" She assured herself that the children understood the meaning of these key terms.

The rest of the lesson involved the teacher's instructing the pupils in applying the rules for forming fractions, and in having them work individually at their desks, practicing representative problems.

### A Japanese Lesson Introducing Fractions[20]

The lesson wasn't formally introduced. Instead, the teacher held up a large beaker containing colored water. "How many liters of juice do you think are in here?," he asked. The pupils offered various guesses. The teacher suggested that they pour the juice into some one-liter beakers.

Each receiving beaker was divided into thirds by horizontal lines. The juice filled one beaker all the way, plus part of a second beaker. The teacher pointed out that, in the second receiving beaker, the juice came up to the lowest line; only one of its three parts was full.

The teacher revealed a second large beaker with colored water, plus two one-liter beakers with lines dividing each into halves. He repeated the pouring procedure, noting that in the second receiving beaker the juice came up to the midline; only one of its two parts was full.

The teacher noted that in the first large beaker, there had been one liter plus one-out-of-three parts of a liter of juice. In the second large beaker, there had been one liter plus one-out-of-two parts of a liter of juice. He wrote 1⅓ and 1½ on the board. Then he asked the pupils to figure out how to represent in writing two parts out of three, two parts out of five, and so forth.

Near the end of the period, the teacher spoke the word "fraction" for the first time. He also introduced the terms "numerator" and "denominator."

This classroom sequence (except for the second juice pouring) can be seen in "The Polished Stones" video beginning at 16 minutes, 20 seconds.

## Specific Strategies of Math Teachers in East Asia

Among the specific strategies of the teachers are these:

- Beginning with "the problem of the day"
- Emphasizing abstract/symbolic reasoning
- Insuring coherence and making connections
- Using formal proofs and deductive reasoning

### Beginning with "The Problem of the Day"

A common feature of mathematics classes in East Asia, including at the secondary level, is that the class begins with the problem of the day.

The problem of the day is not a quiz, drill, review, or homework check. Instead, it's a never-previously-encountered type of problem. It draws on some of what the pupils already know, but it also deliberately exceeds their knowledge, if only slightly.

In Japan,[21] pupils are given up to 15 minutes to tackle the problem individually and/or in spontaneously convened groups. The teacher circulates, answering questions in ways that do not obviate the pupils' need to rack their brains to figure out what to do. Teachers believe that…

> students learn best by first struggling to solve mathematics problems. Frustration and confusion are taken to be a natural part of the learning process, because each person must struggle with a situation or problem first in order to make sense of the information he or she hears later.[22]

In China,[23] the teacher often begins as well by presenting a challenging problem but without necessarily allowing so much time for the pupils to find ways to solve it. The crucial similarity between Japan and China is that in both cases, the challenge introduced by the initial problem often becomes the main focus of the *entire* lesson.

The videotaped sequence of the problem of the day introduced by the geometry teacher in Taiwan is a fine example of this strategy. A variation on this strategy is seen in the Japanese teacher's beakers with colored water.

## Emphasizing Abstract/Symbolic Reasoning

Abstract/symbolic means that, instead of dealing with each new math problem as a unique experience ("concrete"), you begin by trying to recognize the problem as a certain type, i.e., as being similar to a type of problem with which you have experience and know how to solve. To do that, you need to have learned to recognize the *essential features* of math problems, which requires abstract thinking. And you need to see that some details of a new problem can be represented by symbols, which will make its solution far more efficient.

The abilities to abstractly perceive the essential features of a never-before-seen problem, to symbolically represent some of its features, and to select from among several alternatives an appropriate method to solve it, are *high-level thinking skills*. As one research team observed, "These aims cannot be achieved by rote-drill."[24]

Students in East Asia consistently demonstrate that they have these high-level skills. This can result from only one cause: Teachers prioritize their students' learning to think abstractly and symbolically. The teachers accomplish this through the application of either a conceptual variation or a procedural variation strategy, or both.[25]

**Conceptual Variation:** Mathematical concepts are inherently abstract, but they usually can be introduced in ways that make use of physical objects and visual experiences. The objective is to enable the pupils to mentally connect the abstraction with its real-world embodiments.

Here are two examples: (1) A geometrical concept is "non-coplanar lines," which describes two or more lines that are not on the same plane. The pupils' own desks as well as the classroom's walls and ceiling readily supply examples of contrasting planes and lines. (2) A mathematical concept is "equation," an expression that almost always comprises one or more unknowns and an equal sign. Sometimes teachers introduce several equations (such as $2x = 1$; $4x - 3 = 5$; $x^2 + y^2 = 1$) and encourage pupils to collaborate on finding the common features among them all.

In these and other ways, pupils are helped to recognize the essential features of each concept. They gain the ability, when encountering an unfamiliar problem, to categorize it as being of a certain type, and therefore to be solvable using procedures that they've previously learned.

**Procedural Variation:** We've already seen that, in East Asia, math teachers often begin classes with a challenging problem, then expect class members to come up with a variety of ways of finding the answer. In addition, they sometimes lead a class through a step-by-step process for solving a problem, then through different steps for solving the same problem. They introduce more challenging problems of the same type, focusing on the essential feature that calls for a just-learned procedure. They sometimes even intentionally lead their pupils through an erroneous process for handling a problem, asking them to figure out at what point the process breaks down, and why.

**Insuring Coherence; Making Connections**

At the beginning of Chapter 5 of *A Mirror for Americans*, I noted the astonishing level of effort that teachers in East Asia lavish on their special Lesson Study process. Actually, great effort and care go into their planning of *every* lesson, which accounts for their lesson plans being much longer and far more detailed than those of American teachers. Outcomes include that (a) their teaching of math (and other subjects) gains the quality of coherence within each lesson, and (b) their pupils learn to perceive the connections among mathematical ideas, facts, and procedures.[26]

**Coherence:** Coherence within a lesson is attained when the pupils readily perceive that its activities are all related to each other. That's more likely when only one or two main topics are dealt with. One study compared the number of topics in fifth-grade math classes in Taiwan and the U.S. Each lesson was divided into five-minute segments and the topics discussed within each were counted. In Taiwan, 55% of all segments focused on a single topic; in the U.S., only 17% did.[27]

One reason a tight focus is typical is that the problem of the day becomes the centerpiece of the whole lesson. It's approached from several perspectives, leading to (a) awareness of its essential features and (b) recognition that more than one solution procedure often is applicable. One research team remarked that:

> In Japanese classrooms the students work with few problems consisting of many elements, while in American classrooms the students work with many problems consisting of few elements.[28]

**Connections:** Some researchers have become convinced that "making connections" is a critical factor in explaining any individual's math superiority.[29] This means that the person, when encountering a problem, has learned to notice and take into account its conceptual links, i.e., its connections among mathematical ideas, facts, procedures, and fundamental regularities. Coherence in a lesson makes it more likely that pupils will have opportunities to recognize one or more connections. This appears to be a priority of math teachers in East Asia.

For example, a teacher whose priority is that her pupils will master procedures might ask them simply to graph three equations, then check their work. A teacher whose priority is to help pupils make connections might ask them to "graph these three equations *and* examine the role of the numbers in determining the slope of the three associated lines." After the pupils have time to collaborate, she would lead a whole-class discussion about the connections.[30] The TIMSS 1999 Video Study[31] assessed the U.S. plus five other countries whose eighth-graders' math achievement had tested much higher than that of their U.S. peers.[32] The researchers wanted to understand the main purpose of the problems being given to students.

They posited that any problem could be given to students to focus on (a) using procedures, (b) stating concepts, or (c) making connections. When they studied the videotapes to count only the making-connections problems, they were surprised to find that the U.S. teachers were similar to teachers in the other five countries: They gave fewer making-connections problems than some, but more than others.

So the researchers posited that any given problem could then be *discussed in class* in a way that prioritized one of those three purposes, plus a fourth: (d) giving results only. They studied the videotapes again. When they counted only the problems that initially had been given for the purpose of "making connections," the U.S. data were jaw-dropping: 92% were discussed in terms of giving results or using procedures; 8% were discussed in terms of stating concepts. What was the percentage for making connections? Solve for P: P = 100% – (92% + 8%).[33]

**Using Formal Proofs and Deductive Reasoning**[34]

Mathematics is a body of knowledge that has evolved over millennia, advancing by means of a language of step-by-step reasoning or logical processing known as "deduction,"[35] leading to formal "proofs." A proof occurs when one states an assumption about a mathematical or geometrical relationship, then uses deductive logic to demonstrate, step-by-step, that the assumption is correct.

Formal mathematical language is taught and expected to be used in East Asian classrooms,[36] but rarely in U.S. classrooms. One study found that U.S. teachers wanted pupils to *know about* rules in an informal way. But they didn't expect pupils to view rules as separate entities, nor to formally recite any rule during classroom discussions.

Teachers in East Asia took deductive reasoning, rules, and proofs seriously, holding them up as being foundational to mathematics and insisting that pupils formally recite the applicable rule, fully and accurately, as part of each proof:

Teacher: Good. What's your rationale for that conclusion?

Pupil 1: It's based on the rationale of consistent quotient.

Teacher: Can you say that in detail?

Pupil 1: The quotient will stay consistent if two numbers are multiplied or divided by the same number.

Teacher: Sit down, please. Anything else? You, please.

Pupil 2: The quotient will stay consistent if two numbers are multiplied or divided by the same number *at the same time*.

Teacher: Okay, anything else? You, please.

Pupil 3: Except zero.

Teacher: Good. This is a very important condition. We must pay attention to it.[37]

A Malaysian professor who had the opportunity to pay extended visits to Chinese and U.S. primary schools concluded that in the Chinese classrooms, "getting the correct answer wasn't the main goal. It was why, why not, how, what if, and how do you know."[38]

## East Asian Pedagogy vs. American Best Practices

Perhaps you're curious about the differences between East Asian mathematics pedagogy, as reviewed herein, and American researchers' best practices, as revealed by the *Education Week* magazine.

The differences are few. The similarities are striking. I'd like to share with you a couple of examples from two of the articles in the *Education Week* magazine.

In the first article, “5 Research-Backed Ways to Help Students Catch Up in Math,” we read that:

> Math concepts build on each other, and there’s strong evidence that students need explicit, systematic instruction to understand how foundational concepts connect to each other. Students should have regular opportunities to discuss and justify different approaches to solving problems, and teachers should encourage them to think about the underlying logic of their approaches.[39]

And from, “Math Gets Progressively More Abstract. Here’s How to Help Students Keep Up”:

> Students should practice talking through why they choose a particular approach and the steps they take to solve the problem. Beyond learning the correct answer, students benefit from understanding common misconceptions and thinking through why they came to wrong answers. Students learn more when they dig into problems in sufficient detail to highlight the upcoming steps needed to solve them.[40]

In my view, it is discomfiting to realize that, decade after decade, East Asian math teachers have been teaching in exactly these ways. How they taught began to be discovered by Western researchers during the early 1970s (mathematics teaching wasn’t the only focus of their research, but math got more attention than other subjects). Reports of their findings began appearing in academic journals during the late 70s. “The Polished Stones,” intended for educators and concerned citizens, was released in 1980. Books and articles similarly intended for people who aren’t researchers or scholars became available after that. A few of these titles appear below in the Bibliography.

Only one publication, however, rose into national attention (briefly!): Stevenson & Stigler’s short, accessibly written 1992 book, *The Learning Gap: Why Our Schools Are Failing and What We Can Learn from Japanese and Chinese Education*. It’s as insightful now as it was in 1992.

Could it really be true that American math educators remained indifferent to the examples of effective math teaching that Western researchers had been bringing back from a world region whose students have been outperforming ours in every international comparative test?

To find out more about *A Mirror for Americans: What the East Asian Experience Tells Us about Teaching Students Who Excel*, visit the book’s website at amirrorforamericans.info.

## Bibliography

The details in this article were gleaned from the following sources. If you’re interested in learning more about any of these sources, you’ll find my personal review of virtually all of them at this webpage, amirrorforamericans.info/annotated-bibliography/.

Clark, David, & Li Hua Xu (2008). Distinguishing between mathematics classrooms in Australia, China, Japan, Korea, and the United States through the lens of the distribution of responsibility for knowledge generation: Public oral interactivity and mathematical orality. *ZDM, The International Journal of Mathematics Education, 40* (6). 963–72.

Gu, Lingyuan, Rongjin Huang, & Ference Marton (2004). Teaching with variation: A Chinese way of promoting effective mathematics learning. *How Chinese Learn Mathematics: Perspectives from Insiders*, L. Fan et al., eds. Series on Mathematics Education, Vol. 1, World Scientific, 309–47.

Hess, Robert D., & Hiroshi Azuma (1991). Cultural support for schooling: Contrasts between Japan and the United States. *Educational Researcher, 20* (9). 2–8, 12.

Hiebert, James, and nine coauthors (2005). Mathematics teaching in the United States today (and tomorrow): Results from the TIMSS 1999 Video Study. *Educational Evaluation and Policy Analysis, 27* (2). 111–32.

Kawanaka, Takako, James W. Stigler, & James Hiebert (1999). Studying mathematics classrooms in Germany, Japan, and the United States: Lessons from the TIMSS videotape study. *International Comparisons in Mathematics Education*, G. Kaiser et al., eds. Falmer Press, 86–103.

Lee, Shin-ying (1998). Mathematics learning and teaching in the school context: Reflections from cross-cultural comparisons. *Global Prospects for Education: Development, Culture, and Schooling*, S.G. Paris & H.M. Wellman, eds. American Psychological Association, 45–77.

Leung, Frederick K.S. (2005). Some characteristics of East Asian mathematics classrooms based on data from the TIMSS 1999 video study. *Educational Studies in Mathematics, 60*, 199–215.

Lewis, Catherine C., & Ineko Tsuchida (1998). A lesson is like a swiftly flowing river: How research lessons improve Japanese education. *American Educator, 22* (4), 12–17, 50–52.

Lim, Chap Sam (2007). Characteristics of mathematics teaching in Shanghai, China: Through the lens of a Malaysian. *Mathematics Education Research Journal, 19* (1), 77–88.

Linn, Marcia C., Catherine C. Lewis, Ineko Tsuchida, & Nancy B. Songer (2000). Beyond fourth-grade science: Why do U.S. and Japanese students diverge? *Educational Researcher, 29* (3), 4–14.

Marton, Ference, Amy B.M. Tsui, Pakey P.M. Chik, Po Yuk Ko, & Mun Ling Lo (2004). *Classroom Discourse and the Space of Learning*. Routledge.

Mok, Ida Ah Chee, & Paul Morris (2001). The metamorphosis of the 'virtuoso': Pedagogic patterns in Hong Kong primary mathematics classrooms. *Teaching and Teacher Education, 17* (4), 455–68.

Mok, Ida Ah Chee (2006). Shedding light on the East Asian learner paradox: Reconstructing student-centredness in a Shanghai classroom. *Asia Pacific Journal of Education, 26* (2), 131–42.

Park, Kyungmee, & Frederick K.S. Leung (2006). A comparative study of mathematics textbooks of China, England, Japan, Korea, and the United States. *Mathematics Education in Different Cultural Traditions: A Comparative Study of East Asia and the West*, F.K.S. Leung et al., eds. Springer, 227–38.

Perry, Michelle (2000). Explanation of mathematical concepts in Japanese, Chinese, and U.S. first- and fifth-grade classrooms. *Transcending Stereotypes: Discovering Japanese Culture and Education*, B. Finkelstein et al., eds. Intercultural Press, 96–108.

Schleppenbach, Meg, Michelle Perry, Kevin F. Miller, Linda Sims, & Ge Fang (2007). The answer is only the beginning: Extended discourse in Chinese and U.S. mathematics classrooms. *Journal of Educational Psychology, 99* (2), 380–96.

Sekiguchi, Yasuhiro, & Mikio Miyazaki (2000). Argumentation and mathematical proof in Japan. *International Newsletter on the Teaching and Learning of Mathematical Proof*, January/February issue.

Shimizu, Yoshinori (1999). Aspects of mathematics teacher education in Japan: Focusing on teachers' roles. *Journal of Mathematics Teacher Education, 2*, 107–116.

Stevenson, Harold W., & James W. Stigler (1992). *The Learning Gap: Why Our Schools Are Failing and What We Can Learn from Japanese and Chinese Education*. Simon & Schuster.

Stigler, James W., & Harold W. Stevenson (1991). How Asian teachers polish each lesson to perfection. *American Educator, 15* (1), 12–21, 43–47.

Stigler, James W., & Michelle Perry (1990). Mathematics learning in Japanese, Chinese, and American classrooms. *Cultural Psychology*, J. Stigler, et al., eds. Cambridge University Press, 328–53.

Stigler, James W., Clea Fernandez, & Makoto Yoshida (1998). Cultures of mathematics instruction in Japanese and American classrooms. *Teaching and Learning in Japan*, T.P. Rohlen & G.K. LeTendre, eds. Cambridge University Press, 213–47.

Stigler, James W., & James Hiebert (1999). *The Teaching Gap: Best Ideas from the World's Teachers for Improving Education in the Classroom*. The Free Press.

Stigler, James W., & James Hiebert (2004). Improving mathematics teaching. *Educational Leadership, 61* (5), 12–17.

Usui, Hiroshi (1996). Differences in teacher classroom behaviors in the U.S.A. and Japan: A field note. *Research and Clinical Center for Child Development Annual Report, 18 (3)*. Faculty of Education, Hikkaido University, 63–85.

Wang, Tao, & John Murphy (2004). An examination of coherence in a Chinese mathematics classroom. *How Chinese Learn Mathematics: Perspectives from Insiders*, L. Fan et al., eds. Series on Mathematics Education, Vol. 1, World Scientific, 107–23.

Zhang, Dianzhou, Shiqi Li, & Ruifen Tang (2004). The "Two Basics": Mathematics teaching and learning in mainland China. *How Chinese Learn Mathematics: Perspectives from Insiders*, L. Fan et al., eds. Series on Mathematics Education, Vol. 1, World Scientific, 189–201.

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[1] *Education Week*, Vol. 44, Issue 26, May 14, 2025. This publication appears as a magazine, not a tabloid newspaper.

[2] Unlike *Kappan* (full name, *Phi Delta Kappan*), *Education Week* articles rarely includes bibliographies or endnotes.

[3] International comparative testing began around 1970. Here are recent examples of how the average mathematics proficiency scores of American students compare with the scores of their peers in East Asia and Singapore.

- 4th graders, 2019, TIMSS: **U.S. 535**; Singapore 625; Hong Kong (China) 602; Korea 600; Taiwan 599; Japan 593
- 8th graders, 2019, TIMSS: **U.S. 515**; Singapore 616; Hong Kong (China) 578; Korea 607; Taiwan 612; Japan 594
- 15-year-olds, 2018, PISA: **U.S. 478**; Singapore 569; Beijing et al (China) 591; Korea 526; Taiwan 531; Japan 527
- 15-year-olds, 2022, PISA: **U.S. 465**; Singapore 575; Hong Kong (China) 540; Korea 527; Taiwan 547; Japan 536

I found these among the countless comparative tables on the website of our National Center for Educational Statistics at nces.ed.gov. Most significant is the fact that, in the tables, the East Asian students' average mathematics scores are *always* at or very near the top of the rankings, while the U.S. students' average math scores are *always* in the middle.

[4] Cai & Cifarelli (2004), 85–88, 104.

[5] The opinion of Becker et al. (1999), 121–139, is that the fundamental advantage of East Asian math pupils over their American peers is that they learn to be far more sophisticated in how they approach challenging problems. They are more capable of thinking conceptually or abstractly, and of recognizing that there are a variety of ways to solve many problems. See also Cai (2005), 138, who states that "the disparities in the U.S. and Chinese students' problem-solving success rates are related to their use of different strategies." He adds that these disparities have been found even in kindergarten and first-grade children.

[6] Stigler & Stevenson (1991), page uncertain. I was able to acquire only a condensed and unpaginated version of this article from ResearchGate.com. The Japanese lesson is described at the bottom of the fifth page. I edited this account for clarity and brevity.

[7] "The Polished Stones" videotape was made under the supervision of the late Harold W. Stevenson, arguably the leading researcher of educational differences among various locations in East Asia and the United States.

[8] For the section on stance, my sources included Stigler & Stevenson (1991); Stigler et al. (1998), 241–242; Stigler & Hiebert (1999), 66–72; and Lim (2007), 77–88.

[9] The "conductor" analogy was proposed by Mok & Morris (2001), 455–468; see the article's final paragraph. The purpose of their research was to show that a progressive educational reform effort in Hong Kong had affected the manner in which primary school lessons were being delivered. The article presents before-and-after data from classroom

observations. Some findings suggest that, after progressivism was introduced, "student-oriented" methods were increasingly in use; for example, "Direct Instruction" slightly decreased. Some findings are ambiguous. Some findings reveal that *Hong Kong pupils always had been actively involved during lessons; for example, the "before reform" figure for whole-class interactive teaching was 89.5% (Table 2)*.

[10] Lesson Study (a.k.a., lesson research) is a process developed in Japan for improving instruction that involves extensive teacher collaboration and observation of each other's classes to gauge the effectiveness of instructional strategies. To learn more, consult Lewis & Tsuchida (1998); and lessonresearch.net.

[11] For the section on errors, my sources included Stigler & Stevenson (1991); Stigler & Hiebert (1999), 92–93; and Lim (2007), 84–85. See also Chapter 3 of my 2017 book, *The Drive to Learn*.

[12] In a culture such as that of the U.S. where academic success is attributed largely to the student's inborn aptitude, an incorrect answer suggests that the student is innately deficient in aptitude, causing loss of face, lowered self-esteem, and reduced motivation to even *try* to learn. In cultures such as those found in East Asia where academic success is attributed largely to the student's own effort, an incorrect answer reveals where more effort is needed. In *The Aptitude Myth*, see Chapter 16, "American Educational Metamorphosis, III: A 'Given' Joins the Establishment." In *The Drive to Learn*, see Figures 3.1 and 3.2 on page 20.

[13] Lim (2007), 85.

[14] For the section on pace and focus, my sources included Stigler et al. (1998), 230–234, especially Figure 4; Stigler & Stevenson (1991); Usui (1996), 63–85; Perry (2000), 192–193, especially Figure 2; and Linn et al. (2000), 4–8.

[15] Perry (2000), 192–193. The numbers of first-grade classes observed were 40 in Japan, 40 in Taiwan, and 80 in the U.S. This comparison had an unusually high level of statistical significance ($p = .0004$).

[16] Hess & Azuma (1991), 2–8, 12. The authors suggest that in the U.S., teachers fear that if they slow down, pupils' attention will be lost; rapid pace is believed to insure student engagement. Rhetorically, Hess & Azuma pose the question as to whether pupils in Japan tolerate the slow pace because they "are cowed into submission by the teachers' towering authority or fear the consequences of breaking the rules?" Their answer: "No researcher has reached this conclusion."

[17] For the section on the nature of classroom talk, my sources included Stigler et al. (1998), 230–234, especially Figure 4; and Schleppenbach et al. (2007), 390–392.

[18] Clarke & Xu (2008) 969–970, especially Figures 3 and 4. Strangely, the three Hong Kong-based classrooms were relatively weak in the discussion of key mathematical terms; two of them had lower counts than both San Diego-based classrooms. The authors offer no explanation for this anomaly.

[19] Stigler & Stevenson (1991), page uncertain; see endnote 6. The description of the American lesson is very short. I added that the teacher had the pupils work at their desks on representative problems, as this is often a feature of American lessons. I also added Hess & Azuma's phrase (referring to U.S. classrooms) "quick and snappy"; see also endnote 16.

[20] Stigler & Stevenson (1991), page uncertain; see endnote 6. I have added paragraph breaks and edited for clarity.

[21] The term "problem of the day" has become associated primarily with Japanese mathematics teaching. For example, see Stigler & Hiebert (1999), 77–80, 90–91; Stigler et al. (1998), 230–231; and Kawanaka et al. (1999), 91–92.

[22] Stigler & Hiebert (1999), 91; edited for brevity.

[23] Accounts of Chinese mathematics lessons don't speak of the "problem of the day" but do note that one problem, initially presented, often introduces a problem type that will be the focus of the entire lesson. See Wang & Murphy (2004), 109; and Mok (2006), 135–136. In Mok's detailed account of a math lesson for seventh graders, she refers to the teacher's first presenting "the situational question," which immediately leads into "the trial activity."

[24] Gu et al. (2004), 311.

[25] My discussion of conceptual and procedural variation is based on Gu et al. (2004), 315–327. The lead author, Linguan Gu, is credited with introducing conceptual and procedural variation to Chinese math teaching beginning in the 1980s, long before the Western-inspired reforms got underway. See also Park & Leung (2006), 247–261.

[26] My discussion of coherence is based on Stigler et al. (1998), 227–228; and Wang & Murphy (2004), 108–116.

[27] Wang & Murphy (2004), 109. Wang and Murphy attribute this finding to Stigler & Perry (1990), 328–353.

[28] Gu et al. (2004), 314. Attributed to Marton et al. (2004, page number not provided).

[29] My discussion of making connections is based on Stigler & Hiebert (2004), 12–17; and Hiebert et al. (2005), 111–132, especially Figures 3 and 5.

[30] This example is based on Hiebert et al. (2005), 119.

[31] The Third International Mathematics and Science Study (TIMSS) 1999 Video Study focused on mathematics teaching in the high-performing nations. Some of the videos in that study, including those filmed in Japan and Hong Kong, are publicly available. These videos are of complete 45- or 50-minute lessons. They all fall outside the scope of *A Mirror for Americans* because only eighth-grade math and geometry lessons were studied. The videos and many supporting materials are freely available at timssvideo.com.

[32] The five countries in which eighth-grade students' math achievement was much higher than that of their peers in the U.S., and which were studied (together with the U.S.) in the TIMSS 1999 Video Study, were Japan, Hong Kong, Australia, the Netherlands, and the Czech Republic. (Switzerland was included in the 1999 Study, but did not participate in this investigation.)

[33] Here are corresponding findings for the other five countries: Considering only those problems that initially had been given for the purpose of "making connections," the subsequent in-class discussions that focused on "making connections" were: Czech Republic 52%, Japan 48%, Hong Kong 46%, Netherlands 37%, Australia 8%. All data from Hiebert et al. (2005), 122, Figure 5. Stigler & Hiebert (2004), 15–16, comment as follows:

> In the United States, teachers implemented none of the making connections problems in the way in which they were intended. Instead, the U.S. teachers turned most of the problems into procedural exercises or just supplied students with the answers to the problems. Our research indicates that the lower achievement of U.S. students cannot be explained by an overemphasis on concepts or understanding. In fact, 8th graders spend most of their time in mathematical classrooms practicing procedures. They rarely spend time engaged in the serious study of mathematical concepts.

[34] My discussion of formal proofs and deductive reasoning is based on Zhang et al. (2004), 198); Kawanaka et al. (1999), 99–100; Lim (2007), 82–84; and Schleppenbach et al. (2007), 387–393.

[35] Deductive reasoning is discussed in my 2013 book, *The Aptitude Myth* on pages 12–13 and 23–24. Deduction is often confused with induction, including by Arthur Conan Doyle, whose character Sherlock Holmes makes many "amazing deductions" that are, in fact, not deductions but fine examples of inductions!

[36] An insightful article by two Japanese professors notes that the format of mathematical and geometrical discussions, or proofs, is straightforwardly argumentative, which in turn puts the culture of formal mathematics into conflict with the culture of Japan, which is harmony oriented. Thus, pupils' disagreements about a mathematical proof can endanger the harmony of the class. Teachers handle such disagreements as "not just a problem between involved children *but instead frame it as a problem facing the whole class: The conflict is shared among the classroom participants and becomes 'our problem.'* All class members are supposed to work together towards resolution of the problem, so that the resolution produces a recovery of harmony in the classroom community." Sekiguchi & Miyazaki (2000), 5; italics added. See also Leung (2005), 199–215. Leung's examination of the TIMSS videotapes revealed that students in East Asia "were exposed to more instructional content. The problems they worked on were set up mainly using mathematical language and, compared with the problems solved by students in other countries, took longer duration to solve and more proof was involved." (This quote was taken from the abstract of Leung's article.)

[37] Schleppenbach et al. (2007), 391–392. This article includes six examples of classroom discourse in East Asia and the U.S.

[38] Lim (2007), 83. The TIMSS 1999 Video Study compared the number of formal proofs that occurred during Japanese and U.S. eighth-grade mathematics classes: Japan, 53.0%; United States, 0.0%. Kawanaka et al. (1999), 99.

[39] Sparks, Sarah D. (2025, May 14). 5 research-backed ways to help students catch up in math. An Educator's Guide to Stronger Math Instruction and Achievement. *Education Week, 44* (26), 5.

[40] Sparks, Sarah D. (2025, May 14). Math gets progressively more abstract. Here's how to help students keep up. An Educator's Guide to Stronger Math Instruction and Achievement. *Education Week, 44* (26), 7.